VI. *A general Method of describing* Curves, *by the Intersection of* Right-Lines; *moving about Points in a given* Plane. *In a Letter to Dr.* Hoadly, *by the Rev. Mr.* Braikenridge.

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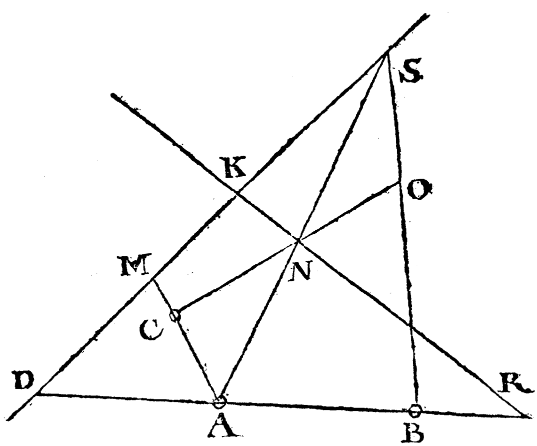
CUM plurimum delectaris Matheseos studio, tantosque progressus in ea ipse feceris, haud ingratum tibi me facturum duxi, si nova quædam de descriptione Curvarum tibi mitterem, quæ a te si probata fuerint, procul dubio & sana & utilia existimabuntur. Habes hic ni fallor Generalem Methodum Lineas, cujuscunque ordinis describendi, ope intersectionum rectarum circa polos revolventium; quæ est *Neutoniana* multo simplicior, & quæ plurima problemata soluta dabit inventu difficillima; ac nescio an ex aliis principiis inveniri queant. Hujus Methodi particularem tantum casum explicatum dedi in Exercitatione illa Geometrica *Londini* edita anno, 1733. Illo tempore rem totam exponere nec commodum, nec aptum censui, quamvis Methodum bene cognitam haberem. Abhinc enim triennium est ex quo in Theorema Generale incideram, sed celare multa me moverunt; et mecum statui, ut biennium saltem peractum esset ab edita illa Exercitatione antequam hæc Generalis Methodus in lucem prodiret. Nihil enim dubitabam, si qui alii hujus Inventi potirentur, quin, particulari casu edito, occasionem arrepturi essent

præſertim laceſſiti, generalem Methodum edendi, ſi talem ſibi compertam revera habuiſſent. Sed ne nimius ſim, en Methodum ipſam pluribus Propoſitionibus expreſſam, quarum demonſtrationes minutim tibi exhibere haud neceſſarium duco; ipſe enim perſpicacius hæc videbis quam ut prolixo calculo aut arduo ratiocinio indigeas. Harum prior demonſtratur in Exercitatione ſupra dicta; & in eum finem huc tranſcripta eſt, ut uno intuitu videas eam contineri ſub hac generali Methodo.

Circa tria puncta data A, B, C, in plano quovis tanquam polos moveantur tres rectæ AN S, BOS, CNO quæ ſeſe interſecent in punctis S, N, O, & ducantur duo interſectionum puncta S, & N, per rectas D K S,

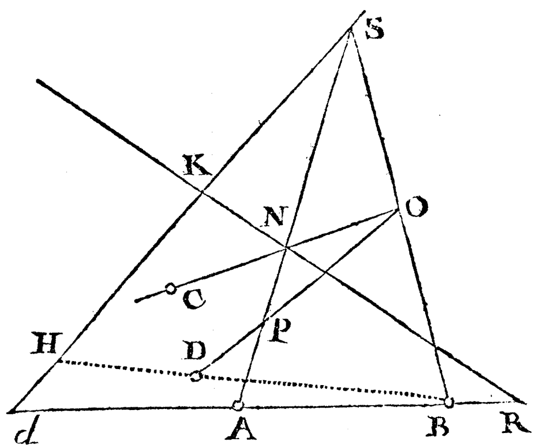


RNK poſitione datas; reliquum O Sectionem Conicam deſcribet. Demonſtratur in *Exerc. Geom. Prop.* I.
Si per puncta A, B, C, agantur rectæ A B, A C ſibi invicem occurrentes in A & rectis R K, D K poſitione datis in R & M; Figura deſcripta tranſibit per

quinque

quinque puncta, B, C, K, M, R. Et hinc patet nova methodus Sectionem Conicam describendi per quinque puncta data omnibus hactenus inventis multo facilior. Vid. *Exerc. Geom. Prop.* 3.

Moveantur circa quatuor puncta A, B, C, D, in plano quovis tanquam polos totidem rectæ A N S, B O S, C N O, D P O, quarum tres A N S, B O S, C N O sese intersecent in tribus punctis S, N, O, & du-

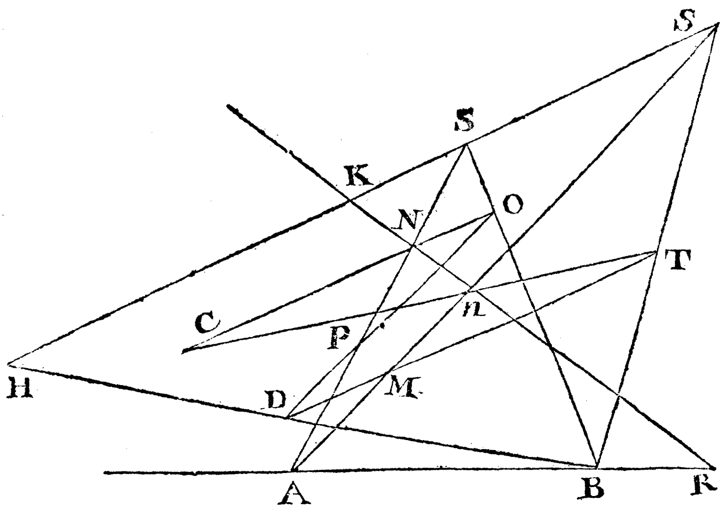


cantur duo intersectionum puncta S, N, per rectas *d* K, R K positione datas, & interea transeat per reliquum O recta D P O ducta a polo quarto D, rectamque A N S secet in P; atque punctum illud P describet Lineam tertii ordinis. Demonstratur ex *undecima Prop. Exerc. Geom.*

Per polos A,B,D, agantur rectæ A B R, B D H sibi occurrentes in B, & rectis K R, K *d* positione datis in R,H ; Figura motu puncti P descripta transibit per quinque puncta A, D, H, K, R, quorum A erit duplex. Hinc deducitur Methodus describendi Lineam

 tertii

tertii ordinis per ſeptem puncta data quorum unum ſit duplex. Dentur enim A, D, H, K, P, M, R, et oportet unum A eſſe duplex. Per duo puncta, H, R, ad aliud K agantur rectæ H K, R K, & jungantur

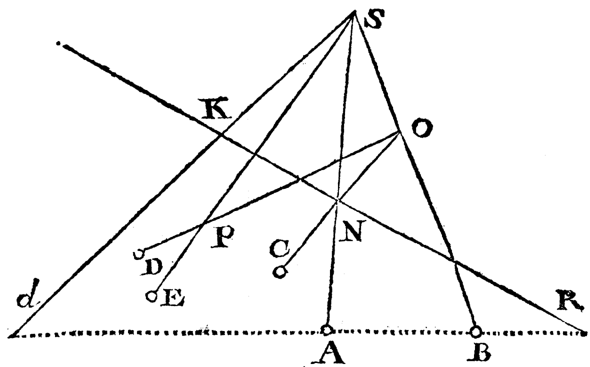


puncta A, R, & H, D, producanturque rectæ AR, H D quæ ſibi occurrant in B. Ductis per A & puncta P, M, rectis A P N S, A M *n s* quæ rectam K R ſecent in N, *n*, rectam vero H K, in S, *s*; per puncta illa S, *s*, ducantur ad B rectæ B S, B *s*, atque per D ad puncta P, M, age rectas D P O, D M T rectis B S, B *s*, occurrentes in O, T. Jungantur puncta O, N, & T, *n*, & producantur rectæ O N, T *n* quæ conveniant in C. Dein circa puncta A, B, C, D, tanquam polos rotentur rectæ A S, BO, C O, D O, quarum tres A S, BO, C O ſeſe interſecent in punctis S, N, O, & ducantur duo S, N, per rectas H K, K R & interea tranſeat ſemper recta DO per reliquum O, quæ rectam ANS ſecet in P,

&

& hæc interſectio P rectarum AS, DO Lineam tertii ordinis deſcribet per ſeptem puncta data, A,D,H,K,M, P,R, tranſeuntem, dupliciter vero per datum A

Deſcribuntur etiam Lineæ tertii ordinis generalius ſed minus commode hoc modo qui & priorem complectitur. Rotentur circa quinque puncta A,B,C, D, E, data tanquam polos totidem rectæ A N S, B O S, C N O,

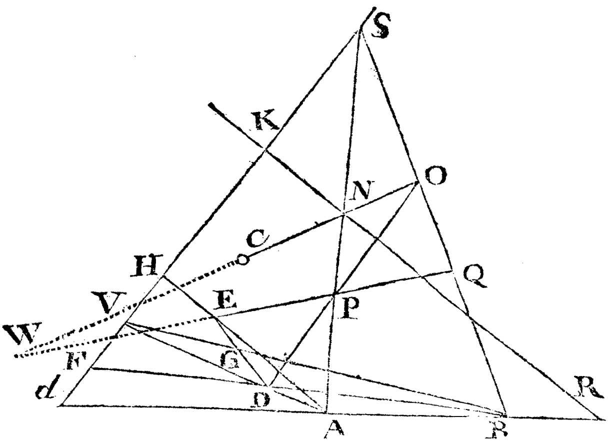


D P O, E P S, quarum tres A N S, B O S, C N O, ſeſe interſecent in punctis N, S, O ; ducantur duo S, N, per rectas poſitione datas *d* K, K R, atque per alterutrum S, e duobus S, N, & reliquum O tranſeant continuo rectæ E P S, D P O per polos E, D, ductæ, quæ conveniant in P : punctum illud P deſcribet lineam tertii ordinis, cum duplice puncto in polo E.

Simili modo deſcribi poſſunt Lineæ quarti ordinis. Moveantur circa quinque juncta data A, B, C, D, E, in plano quovis tanquam polos totidem rectæ A N S, B Q S, C N O, D P O, E P Q, quarum tres, A N S, B Q S, C N O ſibi occurrant in tribus punctis S, N, O ; ducantur duo interſectionum puncta S, N, per rectas *d* K, R K poſitione datas, & interea per reliquum O

tranſeat

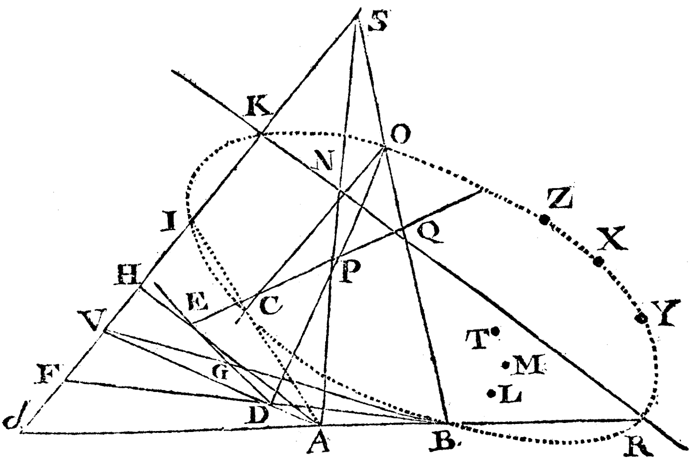
transseat semper recta D P O mobilis circa quartum polum D, quæ secet rectam A N S in P; dein agatur per illud P recta E P Q ducta a polo quinto E, & producatur utrinque ut rectis B Q S, C N O, occurrat



in Q & W: dico puncta Q, W, Lineas quarti ordinis describere. Demonstratur ex *undecima Prop. Exerc. Geom.* Per polos A, E, & B, D, agantur rectæ A E H, B D F, rectæ *d* K positione datæ occurrentes in H, F; Jungantur D, E, atque per polos D, A, ducta A D, rectæ, *d* K occurrente in V; ex illo V educatur recta V B ad polum B quæ rectam D E secet in G. Figura descripta transibit per quinque puncta B, E, G, F, H, tripliciter autem per polum B. Producatur per polos A, B, recta A B R quæ rectæ K R positione datæ occurrat in R; Curva etiam transibit per puncta R, K.

Hinc elicitur methodus ducendi Lineam quarti ordinis per novem puncta data quorum unum sit triplex. Dentur enim B,E,F, G,H,L,M,T,Q, & oportet unum B esse triplex. Jungantur puncta B F, F H, H E, producanturque

ducanturque rectæ BF, FH, HE. Et per puncta EG, GB, agantur rectæ EGD, BGV, quarum EGD rectam BF secet in D, altera vero BGV rectam FH in V. Junctisque V & D ac producta VD



quæ rectæ HE occurrat in A, ducatur per puncta A, B, recta *d*ABR. Dein a punctis B, E, inflectantur ad datum Q rectæ BQS, EPQ, quarum prior BQS conveniat cum FH producta in S; & per puncta A, S, ducta AS occurrente rectæ EQ in P, per illud P ac D producatur recta DPO quæ rectæ BQS occurrat in O: Noteturque punctum O. Et similiter ab iisdem B, E, ad aliud datum T inflectantur rectæ BT*s* E*p*T (supple figuram) quarum BT*s* conveniat cum FH in *s*, & ducta A*s* secante rectam E*p*T in *p*, agatur per *p* & D recta D*p*Z quæ occurrat rectæ BT*s* in Z & notetur Z. Et ita deinceps ducantur rectæ ab iisdem B, E, ad reliqua data M, L, ductisque rectis ab A & D ut prius, notentur puncta inventa XY.

Deinde

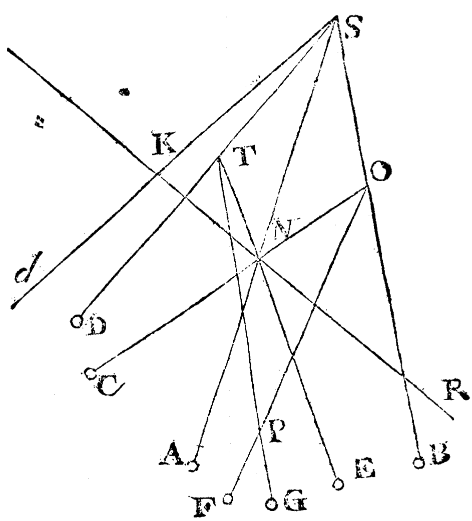
Deinde per quatuor puncta inventa O, Z, X, Y & datum B describatur sectio Conica (vid. *Prop.* 3. *Exerc. Geom.*) quæ rectam F H secet in punctis I, K, rectam vero *d* A B in B, R. Per puncta A, I, agatur recta A I quæ sectionem Conicam secet in I & C; junganturque puncta K, R, & producatur recta K R. Moveantur jam circa quinque puncta A, B, C, D, E, tanquam polos totidem rectæ A S, B S, C N, D O, E Q, quarum tres AS, BS, CN, sibi occurrant in N, S, O, & ducantur concursus N & S rectarum AS, CN, & AS, BS, per rectas KR, FHK, atque interea per polum D & concursum O rectarum B S, C N transeat semper recta D P O quæ rectam A S secet in P; perque illud P & polum E producatur recta E P Q rectam B S secans in Q & hæc intersectio Q rectarum B S, E P describet Lineam quarti ordinis transeuntem per novem data puncta B E F G H L M T Q quorum unum B fiet triplex.

Methodo haud multum dissimili describi potest Linea quarti ordinis per octo puncta data, quorum tria sint duplicia, atque etiam Linea ejusdem ordinis per undecim puncta data, quorum duo sint duplicia, et alia plura hujusmodi. Sed hæc ne nimiam tibi moram injiciam missa faciam: postea tamen explicaturus si non inutilia videantur.

De numero autem punctorum quæ lineam cujuscunque Ordinis determinant compertum habeo, si n sit numerus dimensionum Lineæ erit $n^2 + 1$ numerus punctorum per quæ linea describi potest. *v. g.* Linea secundi ordinis per 5 puncta, tertii per 10, quarti per 17, quinti per 26. Atque hinc deducitur si Linea ordinis n sit multiplici puncto $n - 1$ prædita describi

cribi poteſt per $2n + 1$, *v.g.* Linea tertii ordinis cum duplici puncto, i. e. $n - 1 = 2$ per ſeptem puncta Linea vero quarti ordinis cum triplici puncto per novem, &c. Et generaliter ſi p, q, r, &c. denotent puncta multiplicia quorum numerus ſit m, deſcribi poteſt Curva per $n^2 - p^2 - q^2 - r^2 + m + 1$ puncta, in quibus ſunt m multiplicia, *v.g.* Linea quarti ordinis quæ tria habet duplicia deſcribi poteſt per octo puncta; nam $n = 4$, $p = q = r = 2$, $m = 3$, & $16 - 4 - 4 - 4 + 3 + 1 = 8$.

Eſt & alia methodus a priori non multum abludens deſcribendi Lineas quarti ordinis, ſed paulo complicatior. Moveantur circa ſeptem polos, A, B, C, D, E, F, G, totidem rectæ A S, B S, C N, D S, E N, F O,

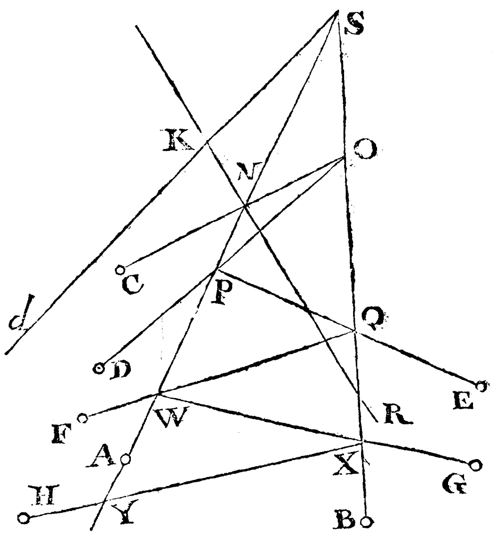


G T quarum una A N S revolvendo ſecet rectas dK, R K, poſitione datas in punctis S, N; ducantur per eorum unum N rectæ C N, E N, & per alterum S rectæ B S, D S, quæ rectis, C N, E N conveniant

 in

in punctis O, T, ut supra Sectiones Conicas describentibus; & interea per eadem O, T, transeant rectæ F O, G T a polis F, G, ductæ quæ sibi occurrant in P; concursus P Lineam quarti ordinis describet cum duplici puncto in utroque polo F & G.

Sed hisce ne diutius immorer, en tandem Theorema Generale. Moveantur circa puncta A,B,C,D,E, F,G,H, &c. quorum numerus sit n, tanquam polos, totidem rectæ AS, BS, CN, DP, EQ, FW, XG, H Y, &c.

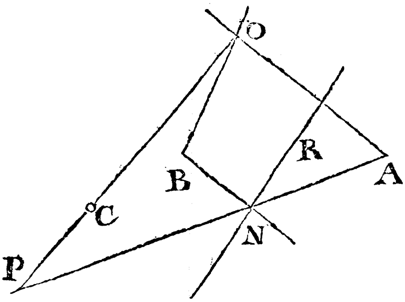


quarum tres A S, B S, C N, sese intersecent in punctis N, S, O, ducantur duo S, N, per rectas dK, K R, positione datas; & interea per reliquum O & polum D transeat recta D P rectam A S secans in P, & per illud P & polum E ducta recta E Q quæ rectam B S secet in Q, & ex hoc Q per polum F agatur F Q rectamque A S secet in W, atque per W & polum G ducta W G rectam B S secante in X, & deinde per X & polum H producatur recta H Y quæ rectæ

S A

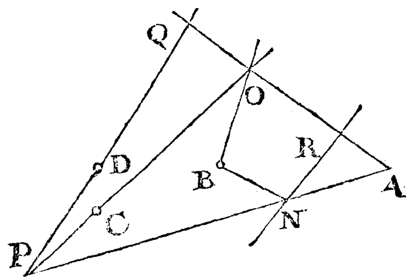
S A occurrat in Y, & ita deinceps; concursus Y rectæ Y H ab ultimo polo H ductæ cum alterutra rectarum A S, B S describet Lineam ordinis $n-1$; & Curva multiplex punctum $n-2$ habebit in polo A vel B prout fuerit descripta concursu rectæ A S aut B S. Puncta O, P, Q, W, X, Y, &c. describent lineas ordinis secundi, tertii, quarti, quinti, sexti, septimi, &c. at si locentur omnes poli A, B, C, D, E, F, G, H, &c. in eadem recta, puncta illa O, P, Q, W, X, Y, &c. etiam totidem rectas describent.

Neutoniana descriptio Curvarum magnopere etiam hac methodo promovetur. Tu satis superque nosti quod si circa data puncta A, B, rotentur dati anguli O A N, O B N, & intersectio N crurum A N, B N, ducatur per rectam N R positione datam; concursum



O crurum A O, B O, Sectionem Conicam describere. Sumatur jam aliud punctum C circa quod moveatur recta O C P, quæ transeat semper per concursum O crurum A O, B O & occurrat alteri cruri A N anguli A in P; concursus P Lineam tertii ordinis describet, dupliciter transeuntem per polum A. Et similiter si concursu cruris B N anguli B describitur curva, ejusdem erit ordinis, & duplex habebit punctum in polo B. Atque hinc etiam patet quomodo describi potest Linea tertii ordinis per septem puncta data quorum unum sit duplex.

Moveantur ut prius anguli O A N, O B N circa data puncta A, B, & per concursum O crurum O A, O B, transeat recta O C P ducta ab alio dato C quæ



cruri A N anguli A occurrat in P, dein per P & quartum datum D agatur recta P D Q cruri A O occurrens in Q ; punctum illud Q describit Lineam quarti ordinis cum triplici puncto in polo A.

Atque ita augendo polorum numerum A, B, C, D, &c. ut sit tandem eorum numerus *n*, Linea descripta erit ejusdem ordinis *n*. Sed notandum si pro angulo O B N substituatur recta quæ moveatur circa polum B, facilior evadet descriptio.

Hactenus Curvæ describuntur tantummodo intersectione rectarum : Quibusdam autem casibus simplicior erit descriptio ope Linearum inferioris ordinis, & de his plurima Theoremata habeo quæ nescio an tua observatione digna sint, præsertim cum multa hujusmodi in Exercitatione supra dicta jam explicantur. Denique enixe rogo ut benigne accipias & quæ minus accurate & inconcinne dicta sint humaniter condones.

VII. *Ex-*